Heterogeneous multiprocessor compositional real-time scheduling

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Compositional hierarchical scheduling frameworks

**Compositionality** property of a system or component which can be analysed by knowing the results of the analysis of its subcomponents (but not their inner details) and how they are combined

**Component** comprises:
- workload
- scheduler
- resource supply
Compositional analysis

(Local) schedulability analysis Analyse schedulability of a component’s workload upon its scheduler and resource supply

Component abstraction Provide abstract representation for the component’s resource demand (hiding the workload’s characteristics) as a single real-time requirement identical to a task (interface)

Interface composition transform set of interfaces abstracting real-time requirements of individual components into an interface abstracting the global requirements of all those components
### Compositional analysis

**Uniprocessor**  
Most work revolves around Mok et al. (2001)’s bounded-delay resource model and Shin and Lee (2003)’s periodic resource model

**Multiprocessor**  
Approaches extending these results include the multiprocessor periodic resource (MPR) model proposed by Shin et al. (2008), Bini et al. (2009a)’s multi supply function (MSF), Bini et al. (2009b)’s parallel supply function (PSF), and Lipari and Bini (2010)’s bounded-delay multipartition (BDM)

None of these works explicitly deals with heterogeneous multiprocessors
Each component $C$ comprises a workload of sporadic tasks, scheduled under GEDF on a cluster of $m'$ identical processors. An MPR $\Gamma = (\Pi, \Theta, m')$ specifies the provision of $\Theta$ units of resource over every period $\Pi$ with concurrency at most $m'$.

**Schedulability test**  Sufficient test based on $\text{sbf}_\Gamma(t)$—the minimum amount of resource that the MPR $\Gamma$ provides over any interval with length $t$.

**Component abstraction**  Pseudo-polynomial algorithm to compute the MPR for $C$ (based on the schedulability test, $\text{sbf}_\Gamma$ replaced by $\text{lsbf}_\Gamma(t)$ for tractability).

**Interface composition**  Transform each MPR interface into a set of $m'$ periodic tasks; if algorithm is GEDF, the union of the task sets can be, in turn, abstracted with MPR.
Multiprocessor periodic resource model: example

Component $C_0$

Gamma $(\Pi, \Theta, 4)$
GEDF

Gamma $(6, 5, 6)$
Gamma $(6, 4, 6)$
Gamma $(8, 3, 8)$
Gamma $(5, 3, 5)$
Gamma $(5, 3, 5)$

Component $C_1$

Gamma $(6, 8.22, 2)$
GEDF

... ... ...

Component $C_2$

Gamma $(8, 2.34, 1)$
GEDF

Gamma $(60, 5, 60)$
Gamma $(100, 5, 100)$

... ... ...

Component $C_3$

Gamma $(6, 8.22, 2)$
GEDF

... ... ...
Open problem

Heterogeneous multiprocessor compositional real-time scheduling

The open problem we here discuss is that of extending virtual cluster-based scheduling to clusters comprising uniform heterogeneous processors, towards compositional hierarchical scheduling frameworks upon heterogeneous multiprocessor platforms.

To the best of our knowledge, there is no literature describing compositional hierarchical scheduling frameworks on heterogeneous multiprocessors.
Heterogeneous multiprocessor periodic resource model

An HMPR model \( \tilde{\Gamma} = (\Pi, \Theta, \pi) \) specifies the provision of \( \Theta \) units of resource over every period of length \( \Pi \) over a virtual cluster \( \pi = \{s'_i\}_{i=1}^{m'} \) comprising \( m' \) heterogeneous processors. Processors are represented as normalized relative speeds, such that \( 1.0 \geq s'_i \geq s'_{i+1} > 0.0, \forall i < m' \).

For the purpose of establishing connections with the work of Shin and Lee (2008), let us note that an MPR \( \Gamma = (\Pi, \Theta, m') \) translates to an HMPR \( \tilde{\Gamma} = (\Pi, \Theta, [s'_i = 1.0]_{i=1}^{m'}) \).
Root component, \( C_0 \), receives a virtual resource provision directly from the physical platform, whereas the remaining components receive their virtual resource provision from \( C_0 \).
Motivation

Related work

Problem

Preliminary intuitions

Open questions

Initial intuition

If we consider only GEDF scheduling, then the results of Shin et al. (2008) up to (and partially including) component abstraction are applicable.

Lemma

Let $\Gamma = (\Pi, \Theta, m')$ be the MPR interface abstracting a component $C$ comprising a task set $\tau$ scheduled under global EDF on a virtual cluster comprising $m'$ identical processors. If $\tau$ is schedulable using $\Gamma$, then $\tau$ is schedulable using any HMPR interface $\tilde{\Gamma} = (\Pi, \Theta, \pi'' = [s''_i]_{i=1}^{m''})$, such that $\sum_{i=1}^{m''} s''_i \geq \lambda_{\pi''} + m'$.
Proof sketch.

The only difference between the considered MPR and HMPR is the virtual platform upon which tasks are scheduled. Let $\pi' = [s_i' = 1.0]_{i=1}^{m'}$ represent the MPR’s platform (on which we know $\tau$ is schedulable) and $\pi'' = [s_i'']_{i=1}^{m''}$, such that $\sum_{i=1}^{m''} s_i'' \geq \lambda_{\pi''} + m'$, the HMPR’s platform. Since these platforms fulfil the conditions of Lemma 1 of Funk et al. (2001):

$$\sum_{i=1}^{m''} s_i'' \geq \lambda_{\pi''} + m' \iff \sum_{i=1}^{m''} s_i'' \geq \lambda_{\pi''} s_1' + \sum_{i=1}^{m'} s_i'',$$

and GEDF is a work-conserving algorithm, $\tau$ is GEDF-schedulable on $\pi''$.\[\square\]
1. This sufficient schedulability condition is too pessimistic; how do we tighten it?

2. Generate optimal HMPR—selection of the optimal $\pi$ brings added complexity
Towards HMPR-specific supply bound function

For MPR $\Gamma = (\Pi, \Theta, m')$:

- $\alpha = \lfloor \Theta / m' \rfloor$ (duration of full VP provision)
- $\beta = \Theta - m' \alpha$ (partial VP provision for 1 time unit)
- $t_1$ is the length of the largest interval with no supply

$sbf_\Gamma(t_1 + 1) = \beta$
Towards HMPR-specific supply bound function

For HMPR $\tilde{\Gamma} = (\Pi, \Theta, \pi = [s_i''']_{i=1}^m)$:

$$\alpha = \lfloor \frac{\Theta}{\sum_{i=1}^m s_i'''} \rfloor; \text{ for } \beta, \text{ two possible approaches:}$$

a) $\beta = \Theta - \alpha \sum_{i=1}^m s_i'''$ (simpler, pessimistic)

b) $\beta = \min \left\{ b \in \text{Sum}_\pi \mid b \geq \Theta - \alpha \sum_{i=1}^m s_i''' \right\}$, \hspace{1cm} $\text{Sum}_\pi = \left\{ \sum_{t \in T} t \mid T \in \mathcal{P}(\pi) \right\}$ (tight, complex)
Example: $\Theta = 2.3$, $\pi = [1.0, 0.6, 0.4]$.

Remember

$$sbf_{\Gamma}(t_1 + 1) = \beta$$

$\alpha = \lfloor 2.3/2.0 \rfloor = 1$; with $\beta$, two possible approaches:

a) $sbf_{\Gamma}(t_1 + 1) = \beta = 2.3 - 2.0 = 0.3$

b) $P(\pi) = \{ [], [0.4], [0.6], [1.0], [0.6, 0.4], [1.0, 0.4], [1.0, 0.6], \pi \}$,
   $\text{Sum}_{\pi} = \{ 0.4, 0.6, 1.0, 1.4, 1.6, 2.0 \}$, $sbf_{\Gamma}(t_1 + 1) = \beta = 0.4$
How do we transform an HMPR interface $\tilde{\Gamma} = (\Pi, \Theta, [s''_{i}]_{i=1}^{m''})$ into $m''$ periodic tasks to be scheduled under GEDF?

Since this transformation must take into account the different relative speeds of the processors in the virtual cluster, how do we guarantee that each of these tasks will, in the end, be scheduled upon the right processor in the physical heterogeneous platform?

How do we compose HMPRs (obtain $\tilde{\Gamma}_0 = \tilde{\Gamma}_1 \oplus \tilde{\Gamma}_2 \oplus \ldots \oplus \tilde{\Gamma}_N$)?
Could other interfaces (MSF, PSF, BDM) bring more advantage in being employed to support heterogeneous multiprocessor platforms?

Could the HMPR be based on a simpler representation of the platform (e.g., only total capacity and $\lambda_{\pi}$ parameter instead of individual processor speeds), in favour of enhanced composability?